Nonlinear relaxation field in charged systems under high electric fields

K. Morawetz
LPC-ISMRA, Boulevard Marechal Juin, 14050 Caen Cedex 5, France
and GANIL, Boulevard Becquerel, 14076 Caen Cedex 5, France
(Received 22 May 2000)

The influence of an external electric field on the current in charged systems is investigated. The results beyond linear response from the classical hierarchy of density matrices are compared with the results from quantum kinetic theory. It is found that even an infinitesimal friction with the background changes the results in a noncontinuous way. The kinetic theory yields a systematic treatment of the nonlinear current beyond linear response. To this end the dynamically screened and field-dependent Lenard-Balescu equation is integrated analytically and the nonlinear relaxation field is calculated. The classical linear response result known as the Debye-Onsager relaxation effect is obtained only if asymmetric screening is assumed. When considering the kinetic equation of one species, the other species have to be screened dynamically while the screening with the same species itself has to be performed statically. Other different approximations are discussed and compared.

PACS number(s): 05.30.−d, 05.20.Dd, 05.60.−k, 72.20.Ht

I. INTRODUCTION

High field transport has become a topic of current interest in various fields of physics. In semiconductors nonlinear transport effects are accessible due to femtosecond laser pulses and shrink devices [1]. In plasma physics these field effects can be studied within such short pulse periods [2]. One observable of interest is the current or the electrical conductivity which gives access to properties of dense nonideal plasmas [3]. In high energy physics transport in strong electric fields is of interest due to pair creation [4]. In order to describe these field effects one can start conveniently from kinetic theory. Within this approach the crucial problem is to derive appropriate kinetic equations that include field effects beyond linear response.

At low strength of the external electric field one expects the linear response regime to be valid. Then the contribution of field effects to the conductivity can be condensed into the Debye-Onsager relaxation effect [5–11], which was first derived within the theory of electrolytes [12–16]. Debye and Hückel gave a limiting law of electrical conductivity [12], which states that the external electric field E on a single charge $Z=1$ is diminished in an electrolyte solution by the amount $E(1+\delta E/E)$, or

$$E^{\mathrm{eff}}=E\left(1-\frac{\kappa e^2}{6T}\right), \tag{1}$$

where e is the elementary charge, E the electric field strength, T the temperature of the plasma, and κ the inverse screening radius of the screening cloud. This law is interpreted as a deceleration force caused by the deformed screening cloud surrounding the charge. Later it was shown by Onsager [13] that this result has to be corrected to

$$E^{\mathrm{eff}}=E\left(1-\frac{\kappa e^2}{3(2+\sqrt{2})T}\right) \tag{2}$$

if the dynamics of ions ($Z=1$) is considered. While the linear response theory seems to reproduce this Onsager result [8,9,11], the kinetic theory seems more to support the Debye result [10,17,11]. The correct treatment is a matter of ongoing debate. In this paper we will give the result beyond linear response for the statically and dynamically screened approximations. We find that even an infinitesimal disturbance like friction with the background leads to a noncontinuous change of the results and reproduces Eq. (2). This can be considered as an example of symmetry breaking of the equations. Different approximations of kinetic theory are discussed and the one that leads to the closest form to the hydrodynamical approximation (Onsager result) is presented.

The kinetic approach describes the time evolution of the one-particle distribution function within an external field $\mathbf{E}$ as

$$\frac{\partial}{\partial t}f-eZ\mathbf{E}\frac{\partial}{\partial \mathbf{k}}f=I[f,\mathbf{E}], \tag{3}$$

where the field-dependent collision integral $I[f,\mathbf{E}]$ has to be provided by different approximations. Integrating this kinetic equation over the momentum $\mathbf{k}$, one obtains the balance of the current. For simplicity we assume that the distribution function can be parametrized by a displaced local equilibrium function with a field- and time-dependent momentum $f(\mathbf{k},t)=f_0(\mathbf{k}-\mathbf{p}(\mathbf{E},t))$, which is related to the current $\mathbf{J}$ as

$$\mathbf{J}(E,t)=nZe\,\frac{\mathbf{p}(E,t)}{m} \tag{4}$$

if the charge is Ze, the density n, and the mass m. The balance equation for the field- and time-dependent local momentum $\mathbf{p}(E,t)$ follows from Eq. (3) as

$$\frac{\partial}{\partial t}\mathbf{p}-eZn\left(1+\frac{\delta E(E,t)}{E}\right)\mathbf{E}=R(E,t)eZn\mathbf{J}(E,t), \tag{5}$$

where the relaxation field $\delta E(E)$ as well as the free resistivity $R(E)$ follow from the field-dependent collision integral. The total conductivity $\sigma\mathbf{E}=\mathbf{J}$ is then given by

$$\sigma(E)=\frac{1+\delta E(E)/E}{R(E)}. \tag{6}$$

The free resistivity R has been the subject of intense investigation in the literature [3]. It is known that the Coulomb divergence for small wave vectors is omitted if screening is included and the divergence at large wave vectors is omitted by the De Broglie wavelength, i.e., by the quantum effects. We will not consider the discussion of the free resistivity R here but concentrate on the relaxation field δE. The free resistivity can be obtained by the same considerations as will be outlined here. We want to point out that the relaxation field will turn out to be free of long wave divergences in the classical limit, in contrast to the free resistivity R.

First we recall the hydrodynamical approach starting from the classical Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy which results in an analytical formula for the classical relaxation effect beyond linear response [18]. This result is then compared with the quantum kinetic approach. We give a short rederivation of the field-dependent kinetic equations in the dynamical screened approximation from the Green's function technique in Sec. III. Two approximations, static screening as well as dynamical screening, are presented. In the fourth section we will derive the field-dependent current analytically. We present both the statically and dynamically screened treatments as analytical results. The classical expression for the statically screened result [17] is compared with the classical result from the hydrodynamical approximation. The dynamical result is then derived analytically also and it is shown that the hydrodynamical result can be approached only for asymmetric screening. In Sec. V we briefly discuss the physical limitation of field strengths for the local equilibrium assumption and the gradient approximation. Section VI summarizes and the Appendix gives the calculation of some involved integrals appearing during the integration of the Lenard-Balescu equation.

II. APPROACH BY CLASSICAL BBGKY HIERARCHY

The starting point for the classical consideration is the BBGKY hierarchy [19,20], which reads for the one-particle distribution function F_a

$$\frac{\partial F_a}{\partial t}+\mathbf{v}\frac{\partial F_a}{\partial \mathbf{r}}+\frac{e_a}{m_a}\overline{\mathbf{E}}\frac{\partial F_a}{\partial \mathbf{v}}-S_aF_a = \sum_b \frac{n_b e_a e_b}{m_a}\frac{\partial}{\partial \mathbf{v}}\int d\mathbf{r}'d\mathbf{v}' F_{ab}(\mathbf{r},\mathbf{r}',\mathbf{v},\mathbf{v}')\frac{\partial}{\partial \mathbf{r}}\frac{1}{|\mathbf{r}-\mathbf{r}'|} \tag{7}$$

and for the two-particle distribution function F_{ab}

$$\begin{aligned}\frac{\partial F_{ab}}{\partial t}&+\mathbf{v}\frac{\partial F_{ab}}{\partial \mathbf{r}}+\mathbf{v}'\frac{\partial F_{ab}}{\partial \mathbf{r}'}+\frac{e_a}{m_a}\overline{\mathbf{E}}\frac{\partial F_{ab}}{\partial \mathbf{v}}\\&+\frac{e_b}{m_b}\overline{\mathbf{E}}\frac{\partial F_{ab}}{\partial \mathbf{v}'}-S_aF_{ab}-S_bF_{ab}\\&=e_ae_b\frac{\partial}{\partial \mathbf{r}}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\left(\frac{1}{m_a}\frac{\partial F_{ab}}{\partial \mathbf{v}}-\frac{1}{m_b}\frac{\partial F_{ab}}{\partial \mathbf{v}'}\right)\\&+\sum_c n_ce_c\int d\mathbf{r}''d\mathbf{v}''\left(\frac{e_a}{m_a}\frac{\partial}{\partial \mathbf{r}}\frac{1}{|\mathbf{r}-\mathbf{r}''|}\cdot\frac{\partial F_{abc}}{\partial \mathbf{v}}\right.\\&\left.+\frac{e_b}{m_b}\frac{\partial}{\partial \mathbf{r}'}\frac{1}{|\mathbf{r}'-\mathbf{r}''|}\cdot\frac{\partial F_{abc}}{\partial \mathbf{v}'}\right)\end{aligned} \tag{8}$$

with the external field $\mathbf{E}$. S_a describes a collision integral with some background which we will specify later. This hierarchy is truncated, approximating as [20]

$$\begin{aligned}F_{ab}&=F_aF_b+g_{ab},\\F_{abc}&=F_aF_bF_c+F_ag_{bc}+F_bg_{ac}+F_cg_{ab},\end{aligned} \tag{9}$$

where $g_{ab}(\mathbf{r}_a,\mathbf{r}_b,\mathbf{v}_a,\mathbf{v}_b)$ is the two-particle correlation function.

Within the local equilibrium approximation we suppose a stationary (for example, a local Maxwellian) distribution for the velocities in the one- and two-particle distribution functions,

$$\begin{aligned}f_a(\mathbf{r},\mathbf{v},t)&=n_a(\mathbf{r},t)\left(\frac{m_a}{2\pi T}\right)^{3/2}\exp\left[-\frac{m_a(\mathbf{v}-\mathbf{u}_a)^2}{2T}\right],\\g_{ab}(\mathbf{r},\mathbf{r}',\mathbf{v},\mathbf{v}',t)&=F_{ab}-F_aF_b\\&=h_{ab}(\mathbf{r},\mathbf{r}',t)\left(\frac{m_am_b}{4\pi^2T^2}\right)^{3/2}\\&\times\exp\left[-\frac{m_a(\mathbf{v}-\mathbf{w}_{ab})^2}{2T}-\frac{m_b(\mathbf{v}'-\mathbf{w}_{ba})^2}{2T}\right].\end{aligned} \tag{10}$$

Here we have introduced the local one-particle density and the local average velocity,

$$\begin{aligned}n_a(\mathbf{r},t)&=\int d\mathbf{v}F_a(\mathbf{r},\mathbf{v},t),\\\mathbf{u}_a&=\frac{1}{n_a}\int d\mathbf{v}\mathbf{v}F_a(\mathbf{r},\mathbf{v},t),\end{aligned} \tag{11}$$

as well as the pair correlation function and the average pair velocity,

$$\begin{aligned}h_{ab}(\mathbf{r},\mathbf{r}',t)&=\int d\mathbf{v}\,d\mathbf{v}'g_{ab}(\mathbf{r},\mathbf{r}',\mathbf{v},\mathbf{v}',t),\\w_{ab}(\mathbf{r},\mathbf{r}',t)&=\frac{1}{h_{ab}}\int d\mathbf{v}\,d\mathbf{v}'\mathbf{v}g_{ab}(\mathbf{r},\mathbf{r}',\mathbf{v},\mathbf{v}',t).\end{aligned} \tag{12}$$

Further on, we suppose that the particles interact with some background (e.g., neutral species or electrolyte solvent) through collision integrals S_a with the following properties:

$$\int d\mathbf{v} S_a f_a = 0,$$

$$\int d\mathbf{v}\mathbf{v} S_a f_a = \frac{1}{b_a m_a}\rho_a \mathbf{u}_a, \tag{13}$$

$$\int d\mathbf{v}\mathbf{v} S_a g_{ab}(\mathbf{r},\mathbf{r}',\mathbf{v},\mathbf{v}',t) = \frac{1}{b_a m_a} h_{ab}\mathbf{w}_{ab},$$

where b_a is the mobility of particles of type a. This friction with the background serves here to couple the two-particle equations and will be considered infinitesimally small at the end. However, as we will demonstrate, this yields a symmetry breaking in the system, which leads to essentially different results than if this friction is neglected.

Fourier transforming the resulting two equations (9) into momentum space and assuming a homogeneous density $n(\mathbf{r})=n$, we arrive at the coupled equation system

$$-\frac{e_a}{T}\overline{\mathbf{E}}\cdot(\mathbf{v}_a-\mathbf{u}_a)f_a = \frac{\mathbf{u}_a}{b_a m_a} + \sum_b \frac{4\pi n_b e_a e_b}{T} \times \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{i\mathbf{k}\cdot(\mathbf{v}_a-\mathbf{w}_{ab})}{k^2} \times f_a(\mathbf{v}_a-\mathbf{w}_{ab}+\mathbf{u}_a)h_{ab}(\mathbf{k}) \tag{14}$$

and

$$\begin{aligned} &i\mathbf{k}\cdot(\mathbf{v}_a-\mathbf{v}_b)g_{ab} - \left[e_a(\mathbf{v}_a-\mathbf{w}_{ab}) + e_b(\mathbf{v}_b-\mathbf{w}_{ba})\right]\frac{\overline{\mathbf{E}}}{T}g_{ab} \\ &= -ie_a e_b \frac{4\pi}{k^2}\mathbf{k}\cdot(\mathbf{v}_a-\mathbf{u}_a-\mathbf{v}_b+\mathbf{u}_b)f_a f_b \\ &\quad - i\int \frac{d\overline{\mathbf{k}}}{(2\pi)^3}\frac{4\pi e_a e_b}{Tk^2}\overline{\mathbf{k}}\cdot(\mathbf{v}_a-\mathbf{w}_{ab}-\mathbf{v}_b+\mathbf{w}_{ba})g_{ab} \\ &\quad \times(\mathbf{k}-\overline{\mathbf{k}}) - \sum_c n_c \int d\mathbf{v}_c \frac{4\pi i e_c}{Tk^2}\left[e_a\mathbf{k}\cdot(\mathbf{v}_a-\mathbf{u}_a)\right. \\ &\quad \left.\times f_a g_{cb}(\mathbf{k}) - e_b\mathbf{k}\cdot(\mathbf{v}_b-\mathbf{u}_b)f_b g_{ac}(\mathbf{k})\right] + S_a g_{ab} + S_b g_{ab} \end{aligned} \tag{15}$$

with

$$\overline{\mathbf{E}} = \mathbf{E} - \sum_b n_b e_b \int d\mathbf{r}_b d\mathbf{v}_b \frac{\partial}{\partial \mathbf{r}_b}\frac{1}{|\mathbf{r}_a-\mathbf{r}_b|}F_b. \tag{16}$$

By multiplying the above equation system by $1, \mathbf{v}_a, \mathbf{v}_b$, and integrating over the velocities we obtain the Onsager equation [13]

$$b_a\left[Th_{ab}(\mathbf{k})\left(1+i\frac{e_a}{e}a\right)+e_a\Phi_b(-\mathbf{k})\right] = -b_b\left[Th_{ab}(\mathbf{k})\left(1-i\frac{e_b}{e}a\right)+e_b\Phi_a(\mathbf{k})\right] \tag{17}$$

with

$$\begin{aligned} k^2\Phi_a(\mathbf{k}) &= 4\pi e_a + \sum_c n_c e_c h_{ac}(\mathbf{k}), \\ k^2\Phi_a(-\mathbf{k}) &= 4\pi e_a + \sum_c n_c e_c h_{ca}(\mathbf{k}) \end{aligned} \tag{18}$$

for the two-particle correlation function h_{ab}. Here we use

$$a = \frac{e\mathbf{k}\cdot\overline{\mathbf{E}}}{k^2 T}. \tag{19}$$

Let us remark here that the friction with background described by the mobilities b couples the two sides of Eq. (17). If we had not considered this friction, $S_i=0$, we would have obtained the result that the left and right hand sides of Eq. (17) vanish separately. This leads essentially to different final expressions even for infinitely small friction. There is no continuous transition between these two extreme cases, pointing to a symmetry breaking between the two treatments. Let us first discuss the case with background friction.

A. With background friction

The system (17) for electrons, $e_e=e$, and ions, $e_i=-Ze$, with charge Z reads when expanded

$$\begin{aligned} Th_{ee} &= -e\frac{\Phi_e(-\mathbf{k})+\Phi_e(\mathbf{k})}{2}, \\ Th_{ei} &= -e\frac{\Phi_i(-\mathbf{k})-Z(b_i/b_e)\Phi_e(\mathbf{k})}{1+b_i/b_e+ia[1+(b_i/b_e)Z]}, \\ Th_{ie} &= -e\frac{\Phi_i(\mathbf{k})-Z(b_i/b_e)\Phi_e(-\mathbf{k})}{1+b_i/b_e-ia[1+(b_i/b_e)Z]}, \\ Th_{ii} &= Ze\frac{\Phi_i(-\mathbf{k})+\Phi_i(\mathbf{k})}{2}. \end{aligned} \tag{20}$$

This we can solve together with Eq. (18). First we calculate the effective field strength at the position of the electron in linear response (the Onsager result [13]),

$$\frac{\delta E}{E}\mathbf{E} = -i\frac{\mathbf{E}}{E}\frac{1}{(2\pi)^2}\int_0^\infty k^3 dk\int_{-1}^1 d(\cos\theta)\cos\theta\ \Phi_e(\mathbf{k}) = \mathbf{E}\frac{\kappa e^2}{3T}\frac{Zq}{\sqrt{q}+1} \tag{21}$$

with $\kappa^2=\kappa_e^2(1+Z)=4\pi(e^2 n_e+Z^2e^2n_i)/T$ and

$$q = \frac{b_e+Zb_i}{(1+Z)(b_e+b_i)}. \tag{22}$$

For singly charged ions ($Z=1$) the influence of the mobilities drops out and we recover the result (2).

Since this result is independent of the mobilities one could conclude that this is a universal limiting law. However, we will express two doubts here. As one sees for charges $Z>1$ the result (2) is approached only in the limit where the ion mobilities are much smaller than the electron mobilities, $b_i/b_e\to 0$. This means, of course, that the electrons have a different friction with a background than the ions. In other words, there is an explicit symmetry breaking mechanism included by assuming such collision integrals with the background. Therefore we will obtain a different solution if we consider no friction.

The second remark concerns the limit of the one-component system, which one can obtain by setting $Z=-1$. The Onsager result or hydrodynamical result with friction (21) leads to twice the Debye result (1) in this case, but with opposite sign. In contrast, we will see in the following that a perfectly symmetric treatment of the species without friction with a background will lead to a vanishing one-component limit, as it should. This again underlines the symmetry breaking if one assumes an infinitesimal small friction with a background.

For completeness, we want to recall the expression of the nonlinear Onsager result [18,17] that is obtained from the limit $b_i/b_e\to 0$ of the system (20):

$$\begin{aligned}
&Th_{ee}+e\frac{\varphi_e(-\mathbf{k})+\varphi_e(\mathbf{k})}{2}=0,\\
&h_{ei}\left(T+ie\frac{\mathbf{k}\cdot\mathbf{E}}{k^2}\right)+e\varphi_i(-\mathbf{k})=o\left(\frac{b_i}{b_e}\right)=0,\\
&-h_{ie}\left(T-ie\frac{\mathbf{k}\cdot\mathbf{E}}{k^2}\right)-e\varphi_i(\mathbf{k})=o\left(\frac{b_i}{b_e}\right)=0,\\
&Th_{ii}-Ze\frac{\varphi_i(-\mathbf{k})+\tilde{\varphi}_i(\mathbf{k})}{2}=0.
\end{aligned}\tag{23}$$

One obtains [14,18] the result for $Z=1$,

$$\begin{aligned}
\delta\mathbf{E}&=-\frac{e^2\kappa_e}{3(1+\sqrt{2})T}\mathbf{E}F_H\left(\frac{eE}{T\kappa_e}\right)\\
&=-\frac{e^2\kappa_e}{6T}\mathbf{E}\times\begin{cases}2-\sqrt{2}+o(E)\\ 3\kappa T/2eE+o(1/E)^2\end{cases}
\end{aligned}\tag{24}$$

with

$$\begin{aligned}
F_H(\alpha)=\frac{3(1+\sqrt{2})}{\alpha^2}\Bigg[&\frac{1}{2}\sqrt{\alpha^2+2}-1+\frac{1}{\alpha}\arctan(\alpha)\\
&-\frac{1}{\alpha}\arctan\left(\frac{\alpha}{\sqrt{\alpha^2+2}}\right)\Bigg].
\end{aligned}\tag{25}$$

The numerical values of this result will be discussed in Sec. IV C.

B. Without background

Now we reconsider the steps from Eq. (15) to Eq. (17) without friction with the background. We obtain the result that both sides of Eq. (17) vanish separately,

$$\begin{aligned}
&Th_{ab}(\mathbf{k})\left(1+i\frac{e_a}{e}a\right)+e_a\Phi_b(-\mathbf{k})=0,\\
&Th_{ab}(\mathbf{k})\left(1-i\frac{e_b}{e}a\right)+e_b\Phi_a(\mathbf{k})=0.
\end{aligned}\tag{26}$$

Both equations have identical solutions h_{ab}, which can be easily verified using the symmetry $h_{ab}(\mathbf{k})=h_{ba}(-\mathbf{k})$. Together with Eq. (18) we can solve for Φ_e and the following relaxation field is obtained instead of (24):

$$\delta\mathbf{E}=-e^2\kappa_e\sqrt{\frac{1+Z}{6T}}(Z+1)\mathbf{E}\,F_N\left(\frac{eE}{T\kappa_e}\right).\tag{27}$$

This becomes for $Z=1$

$$\frac{\delta E}{E}=-\frac{e^2\kappa_e}{6T}\times\begin{cases}2+o(E)\\ 3\kappa T/eE+o(1/E)^2\end{cases}\tag{28}$$

with

$$\begin{aligned}
F_N(\alpha)=\frac{3}{(1+Z)\alpha^2}\Bigg[&\sqrt{4+(1+Z)\alpha^2}\\
&+\frac{4}{\sqrt{1+Z}\alpha}\ln\frac{2}{\sqrt{1+Z}\alpha+\sqrt{4+(1+Z)\alpha^2}}\Bigg].
\end{aligned}\tag{29}$$

We see that the linear response result for $Z=1$ is twice the Debye result (1). For the equally charged system $Z=-1$, which would coincide with a one-component plasma, no relaxation effect appears, as one would expect. In other words, in a perfectly symmetric mathematical two-component plasma there is a different relaxation effect than in a system that distinguishes the components by a different treatment of friction. The Onsager result (24) does not vanish for the limit of a one-component plasma $Z=-1$. This is due to the different treatment of ions and electrons there, which explicitly assumes a two-component plasma. Therefore the limit $Z=-1$ does not work there.

This result is quite astonishing. One would expect that the limiting procedure that transforms the system (18) into (26) would also lead to a smooth transitions of the end results. However, this is not the case. While the separate limit of $b_{e,i}\to\infty$ of Eq. (18) leads to Eq. (26) there is no possibility of transforming the result (21) into the linear response result of (28). This underlines that due to the (even infinitesimally small) friction assumed in obtaining Eq. (21) there occurs a symmetry breaking in the sense that the electrons and ions are no longer symmetrically treated.

We have to keep this lesson in mind when we now advance and investigate the systematic treatment by quantum kinetic theory. There we will also find completely different results when we use asymmetric screening compared to symmetric screening. Of course, we will not assume any phenomenological friction since kinetic theory provides for a

systematic description of all processes occurring. Here we want only to point out that the above symmetry breaking is the main reason for the confusion in literature. Following the linear response formalism an asymmetric treatment of two-particle correlation functions is used in that the electrons are statically screened [11]. This seemingly innocent usage leads there to an occasional agreement for $Z=1$ with the Onsager result (2).

We want to point out here another advantage of the kinetic theory. The classical local equilibrium or hydrodynamical approximation does not lead to a mass dependence of the relaxation effect. This will be provided by kinetic theory.

III. QUANTUM KINETIC THEORY

We will formulate the kinetic theory within gauge invariant functions including all field effects. The most promising theoretical tool is the Green's function technique [1,21–23]. The resulting equations show some typical deviations from the ordinary Boltzmann equation. (i) The collision broadening consists in a smearing out of the elementary energy conservation of scattering. This is necessary to ensure global energy conservation [24]. (ii) The intra-collisional field effect, gives additional retardation effects in the momentum of the distribution functions. This comes mainly from the gauge invariance.

One of the most important questions is the range of applicability of these kinetic equations. Up to which field strengths are such modifications important and appropriately described within one-particle equations? In [25] this question was investigated for semiconductor transport. It was found that for high external fields the intracollisional field effect becomes negligible. This range is given by a characteristic time scale of field effects $\tau_F^2=m\hbar/(e\mathbf{E}\cdot\mathbf{q})$ which has to be compared with the inverse collision frequency. This criterion is a pure quantum one. It remains a question whether there are also criteria in the classical limit. For a plasma system we will show in Sec. V that there is indeed a critical value of the field strength which can be given by classical considerations.

A. Definitions

In order to describe correlations in highly nonequilibrium situations, we define various correlation functions by different products of creation and annihilation operators:

$$\begin{aligned} G^{>}(1,2)&=\langle\Psi(1)\Psi^{\dagger}(2)\rangle \\ G^{<}(1,2)&=\langle\Psi^{\dagger}(2)\Psi(1)\rangle. \end{aligned} \tag{30}$$

Here $\langle\ \rangle$ is the average value with the unknown statistical *nonequilibrium* operator ρ and 1 denotes the cumulative variables $(\mathbf{r}_1,s_1,t_1,\ldots)$ of space, spin, time, etc. The equation of motion for the correlation functions is given in the form of the Kadanoff-Baym equation [26,27,23]

$$\begin{aligned} &-i(G_0^{-1}G^{<}-G^{<}G_0^{-1}) \\ &\quad=i(G^R\Sigma^{<}-\Sigma^{<}G^A)-i(\Sigma^RG^{<}-G^{<}\Sigma^A), \end{aligned} \tag{31}$$

where the retarded and advanced functions are introduced as $A^R(1,2)=-i\Theta(t_1-t_2)[A^{>}\pm A^{<}]$ and $A^A(1,2)=i\Theta(t_2-t_1)[A^{>}\pm A^{<}]$. Here operator notation is employed, where products are understood as integrations over intermediate variables (time and space) and the upper (lower) sign stands for fermions (bosons). The Hartree-Fock drift term reads

$$G_0^{-1}(11')=\left(i\hbar\frac{\partial}{\partial t_1}+\frac{\hbar^2}{2m}\nabla_{\mathbf{x}_1}^2-\Sigma_{HF}(11')\right)\delta(1-1') \tag{32}$$

with $\Sigma_{HF}(1,1')$ the Hartree-Fock self-energy.

B. Gauge invariance

In order to have an unambiguous method of constructing approximations we need to formulate our theory in a gauge invariant way. This can be done following a procedure known from field theory [28]. This method was applied to high field problems in [29]. With the help of the Fourier transform of an arbitrary function $G(x,X)$ over the relative coordinates $x=(\mathbf{r}_2-\mathbf{r}_1,t_2-t_1)=(\mathbf{r},\tau)$ with the center of mass coordinates $X=((\mathbf{r}_2+\mathbf{r}_1)/2,(t_2+t_1)/2)=(\mathbf{R},t)$, one can introduce a gauge invariant Fourier transform of the difference coordinates x,

$$\begin{aligned} \overline{G}(k,X)=&\int dxG(xX) \\ &\times\exp\left[\frac{i}{\hbar}\int_{-1/2}^{1/2}d\lambda x_\mu\left(k^\mu+\frac{e}{c}A^\mu(X+\lambda x)\right)\right]. \end{aligned} \tag{33}$$

For constant electric fields, which will be of interest in the following, one obtains a generalized Fourier transform

$$\overline{G}(k,X)=\int dx\, e^{(i/\hbar)[x_\mu k^\mu+e\mathbf{r}\cdot\mathbf{E}t]}G(x,X),$$

where the χ function was chosen in such a way that the scalar potential is zero, $A^\mu=(0,-c\mathbf{E}t)$. Therefore, we have the following rules for formulating the kinetic theory gauge invariantly. (1) Fourier transform of the four-dimensional difference variable x to the canonical momentum p. (2) Shift the momentum to kinematic momentum according to $\mathbf{p}=\mathbf{k}-e\mathbf{E}t$. (3) The gauge invariant functions $\overline{G}$ are then given by

$$G(\mathbf{p},t)=G(\mathbf{k}-e\mathbf{E}t,t)=\overline{G}(\mathbf{k},t)=\overline{G}(\mathbf{p}+e\mathbf{E}t,t). \tag{34}$$

We shall make use of these rules in the following sections. In [30] this procedure was generalized for two-particle Green's functions and leads to the field-dependent Bethe-Salpeter equation.

C. Equation for Wigner distribution

With the help of the gauge invariant formulation of the Green's function (Sec. III B), we can write the kinetic equation for the Wigner function $f(\mathbf{p},t)=G^{<}(\mathbf{p},\mathbf{R},t,\tau=0)$ finally as [31]

$$\frac{\partial}{\partial t}f(\mathbf{k},t)+e\mathbf{E}\nabla_{\mathbf{k}}f(\mathbf{k},t)=\int_0^{t-t_0}d\tau\left[\left\{G^{>}\left(\mathbf{k}-\frac{e\mathbf{E}}{2}\tau,\tau,t-\frac{\tau}{2}\right),\Sigma^{<}\left(\mathbf{k}-\frac{e\mathbf{E}}{2}\tau,-\tau,t-\frac{\tau}{2}\right)\right\}_{+}\right.$$
$$\left.-\left\{G^{<}\left(\mathbf{k}-\frac{e\mathbf{E}}{2}\tau,\tau,t-\frac{\tau}{2}\right),\Sigma^{>}\left(\mathbf{k}-\frac{e\mathbf{E}}{2}\tau,-\tau,t-\frac{\tau}{2}\right)\right\}_{+}\right]. \tag{35}$$

This kinetic equation is exact in time convolutions. This is necessary because gradient expansions in time are connected with linearization in electric fields and consequently fail [32]. The gradient approximation in space has been applied, assuming slowly varying processes in space, and we have dropped all R dependence for simplicity. This approximation is discussed in the last section, Eq. (91), and corresponds to the limit of a weakly coupled plasma, which we employed already in Sec. II. Please recall that due to the Coulomb gauge we do not have space inhomogeneity in the electric field.

D. The problem of the ansatz

In order to close the kinetic equation (35), it is necessary to know the relation between $G^{>}$ and $G^{<}$. This problem is known as an ansatz and discussed in the literature [31,33,34]. We will use the generalized Kadanoff and Baym ansatz [33] where an expression is given for the $G^{<}$ function in terms of expansion after various times. We can write in Wigner coordinates

$$G^{<}(\mathbf{p},t,\tau)=f\left(\mathbf{p},t-\frac{|\tau|}{2}\right)A(\mathbf{p},\tau,t). \tag{36}$$

This generalized Kadanoff-Baym (GKB) ansatz is an exact relation as long as the self-energy is taken in the Hartree-Fock approximation.

In order to define relation (36) we have to know the spectral function A. The spectral properties of the system are described by the Dyson equation for the retarded Green's function. For free particles and parabolic dispersions, the gauge invariant spectral function [34,35] follows as

$$A_0(\mathbf{k},\omega)=2\int_0^{\infty}d\tau\cos\left(\omega\tau-\frac{k^2}{2m\hbar}\tau-\frac{e^2E^2}{24m\hbar}\tau^3\right)$$
$$=\frac{2\pi}{\epsilon_E}\mathrm{Ai}\left(\frac{k^2/2m-\hbar\omega}{\epsilon_E}\right) \tag{37}$$

where $\mathrm{Ai}(x)$ is the Airy function [36] and $\epsilon_E=(\hbar^2e^2E^2/8m)^{1/3}$. It is instructive to verify that Eq. (37) satisfies the frequency sum rule $\int d\omega A_0(\omega)=2\pi$. The interaction-free but field-dependent retarded Green's function G_o^R can be obtained from the interaction-free and field-free Green's function by a simple Airy transformation [37]. This is an expression of the fact that the solutions of the Schrödinger equation with constant electric field are Airy functions. The retarded functions can therefore be diagonalized within those eigensolutions [38,29]. It can be shown that Eq. (37) remains valid even within a quasiparticle picture [37], where we have to simply replace the free dispersion $k^2/2m$ by the quasiparticle energy ϵ_k.

Together with the requirement of gauge invariance of Sec. III B and using the quasiparticle spectral function (37) with quasiparticle energies ϵ_k instead of $k^2/2m$, the GKB ansatz finally reads

$$G^{<}(\mathbf{k},\tau,\mathbf{r},t)=\exp\left\{-\frac{i}{\hbar}\left(\epsilon_k\tau+\frac{e^2E^2}{24m}\tau^3\right)\right\}$$
$$\times f\left(\mathbf{k}-\frac{e\mathbf{E}|\tau|}{2},\mathbf{r},t-\frac{|\tau|}{2}\right) \tag{38}$$

and analogously for $G^{>}$ by replacing $f\leftrightarrow(1-f)$. In order to get more physical insight into this ansatz one transforms into the frequency representation

$$G^{<}(\mathbf{k},\omega,\mathbf{r},t)=2\int_0^{\infty}d\tau f\left(\mathbf{k}-\frac{e\mathbf{E}\tau}{2},t-\frac{\tau}{2}\right)$$
$$\times\cos\left(\omega\tau-\epsilon(\mathbf{k},\mathbf{r},t)\frac{\tau}{\hbar}-\frac{e^2E^2}{24m\hbar}\tau^3\right). \tag{39}$$

Neglecting the retardation in f one recovers the Kadanoff and Baym ansatz [26] with the spectral function (37). The generalized ansatz takes history into account by an additional memory. This ansatz is superior to the Kadanoff-Baym ansatz in the case of high external fields in several respects [39]: (i) it has the correct spectral properties, (ii) it is gauge invariant, (iii) it preserves causality, (iv) the quantum kinetic equations derived with Eq. (48) coincide with those obtained with the density matrix technique [40,41,31], and (v) it reproduces the Debye-Onsager relaxation effect [10].

Other choices of ansatz can be appropriate for other physical situations. For a more detailed discussion, see [42].

E. Kinetic equation in the dynamically screened approximation

For Coulomb interaction it is unavoidable to consider screening if one does not want to obtain long range (short wave vector) divergences. To obtain an explicit form for the kinetic equation we have to determine the self-energy $\Sigma^{>,<}$. The dynamically screened approximation is given by expressing the self-energy by a sum of all ring diagrams. The resulting kinetic equation is the quantum Lenard-Balescu equation, which was derived for high fields in [37]. We give this approximation in exact time convolutions. The self-energy is given in terms of the dynamical potential $\mathcal{V}$,

$$\Sigma_a^<(\mathbf{k},t,t')=\int \frac{d\mathbf{q}}{(2\pi\hbar)^3}\mathcal{V}_{aa}^<(\mathbf{q},t,t')G_a^<(\mathbf{k}-\mathbf{q},t,t'), \quad (40)$$

where the dynamical potential is expressed within Coulomb potentials $V_{ab}(\mathbf{q})$

$$\mathcal{V}_{aa}^<(\mathbf{q},t,t')=\sum_{dc} V_{ad}(\mathbf{q})\mathcal{L}_{dc}^<(\mathbf{q},t,t')V_{ca}(\mathbf{q}) \quad (41)$$

via the density-density fluctuation

$$\mathcal{L}_{ab}^<(\mathbf{q},t,t')=\delta_{ab}\int d\bar{t}d\bar{\bar{t}}(\mathcal{E}^r)^{-1}(\mathbf{q},t,\bar{t})L_{aa}^<(\mathbf{q},\bar{\bar{t}},\bar{t}) \times(\mathcal{E}^a)^{-1}(\mathbf{q},\bar{\bar{t}},t'). \quad (42)$$

Here L is the free density fluctuation,

$$L_{aa}^<(\mathbf{q},t,t')=\int \frac{d\mathbf{p}}{(2\pi\hbar)^3}G_a^<(\mathbf{p},t,t')G_a^>(\mathbf{p}-\mathbf{q},t',t), \quad (43)$$

and $\mathcal{E}^{r/a}$ the retarded/advanced dielectric function,

$$\mathcal{E}^{r/a}(\mathbf{q},t,t')=\delta(t-t')\pm i\Theta[\pm(t-t')]\sum_b V_{bb}(\mathbf{q}) \times[L^>(\mathbf{q},t,t')-L^<(\mathbf{q},t,t')]. \quad (44)$$

One can easily convince oneself that this set of equations (40)–(44) is gauge invariant.

We can directly introduce this set of equations into the equation for the Wigner function (35) and obtain after some algebra for the kinetic equation

$$\frac{\partial}{\partial T}f_a+e\mathbf{E}\frac{\partial}{\partial \mathbf{k}_a}f_a=I_a^{\rm in}(\mathbf{k},t)-I_a^{\rm out}(\mathbf{k},t). \quad (45)$$

The collision-in integral is

$$I_a^{\rm in}(\mathbf{k},t)=2\sum_b \int \frac{d\mathbf{q}}{(2\pi\hbar)^3}V_{ab}^2(\mathbf{q})\int_0^\infty d\tau\int\frac{d\omega}{2\pi} \times\cos\left[(\epsilon_{k-q}^a-\epsilon_k^a-\omega)\tau+\frac{e_a\mathbf{E}\cdot\mathbf{q}\tau^2}{2m_a}\right] \times f_a(\mathbf{k}-\mathbf{q}-e_a\mathbf{E}\tau,t-\tau)[1-f_a(\mathbf{k}-e_a\mathbf{E}\tau,t-\tau)] \times\frac{L_{bb}^<\left(\mathbf{q},\omega,t-\frac{1}{2}\tau\right)}{\left|\mathcal{E}\left(\mathbf{q},\omega,t-\frac{1}{2}\tau\right)\right|^2} \quad (46)$$

with the free density fluctuation (43)

$$L_{bb}^<(\mathbf{q},\omega,t)=-2\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\int_0^\infty d\tau \times\cos\left[(\omega-\epsilon_p^b+\epsilon_{p+q}^b)\tau+\frac{e_b\mathbf{E}\cdot\mathbf{q}\tau^2}{2m_b}\right] \times f_b\left(\mathbf{p}+\mathbf{q},t-\frac{1}{2}\tau\right)\left[1-f_b\left(\mathbf{p},t-\frac{1}{2}\tau\right)\right] \quad (47)$$

and $I^{\rm out}$ is given by $f\leftrightarrow 1-f$ and $L^<\leftrightarrow L^>$. Here we used the ansatz (38) and have employed the approximation $t\pm\frac{1}{2}\tau\approx t$ in the density fluctuation (42), which corresponds to a gradient approximation in time for the density fluctuations. Since the center-of-mass time dependence is carried only by the distribution functions in Eq. (42), this approximation is exact in the quasistationary case which we investigate in the next section. All internal time integrations remain exact. Of course, for time-dependent phenomena we have to question this approximation.

Equation (46) represents the field-dependent Lenard-Balescu kinetic equation [37], which was here slightly rewritten in a form that will turn out to be very convenient for the later analytical integration. Other standard approximations like the T-matrix [30] approximation resulting in a field-dependent Bethe-Salpeter equation can be used.

Kinetic equation in a statically screened approximation

While we will give analytical results for the dynamical screened approximation (46) it is useful to see the limit of static screening too. Using the static approximation for the dielectric function $\mathcal{E}(\mathbf{q},0,t)$ in Eq. (46), the kinetic equation for statically screened Coulomb potentials in high electric fields appears as [10,31,37]

$$\frac{\partial}{\partial T}f_a+e\mathbf{E}\frac{\partial}{\partial\mathbf{k}_a}f_a=\sum_b I_{ab}, \quad (48)$$

$$I_{ab}=\frac{2(2s_b+1)}{\hbar^2}\int\frac{d\mathbf{k}_a'd\mathbf{k}_bd\mathbf{k}_b'}{(2\pi\hbar)^6}\delta(\mathbf{k}_a+\mathbf{k}_b-\mathbf{k}_a'-\mathbf{k}_b') \times\{f_{a'}f_{b'}(1-f_a)(1-f_b)-f_af_b(1-f_{a'})(1-f_{b'})\} \times V_s^2(\mathbf{k}_a-\mathbf{k}_a')\int_0^\infty d\tau\cos\left\{(\epsilon_a+\epsilon_b-\epsilon_a'-\epsilon_b')\frac{\tau}{\hbar} -\frac{\mathbf{E}\tau^2}{2\hbar}\left(\frac{e_a\mathbf{k}_a}{m_a}+\frac{e_b\mathbf{k}_b}{m_b}-\frac{e_a\mathbf{k}_a'}{m_a}-\frac{e_b\mathbf{k}_b'}{m_b}\right)\right\}$$

with $f_b=f_b(k_b-e_bE\tau,T-\tau)$. The potential is the static Debye potential

$$V_s(p)=\frac{4\pi e_ae_b\hbar^2}{p^2+\hbar^2\kappa^2} \quad (49)$$

and the static screening length κ is given by

$$\kappa^2=\sum_c\frac{4\pi e_c^2n_c}{T_c} \quad (50)$$

in the equilibrium and nondegenerate limit. Here T_c is the temperature of species c, with charge e_c, spin s_c, and mass m_c.

If we had used the conventional Kadanoff and Baym ansatz ([26]) we would have obtained a factor 1/2 in different retardations [31]. This would lead to no relaxation effect at all [10]. Furthermore, it is assumed that no charge or mass transfer will occur during the collision. Otherwise one would obtain an additional term in the cosine function proportional to τ^3.

Two modifications of the usual Boltzmann collision integral can be deduced from Eq. (48). (i) A broadening of the δ distribution function of the energy conservation and an additional retardation in the center-of-mass times of the distribution functions. This is known as collisional broadening and is a result of the finite collision duration [43]. This effect can be observed even if no external field is applied. It is interesting to note that this collisional broadening ensures the conservation of the total energy [24]. If this effect is neglected one obtains the Boltzmann equation for the field-free case. (ii) The electric field modifies the broadened δ distribution function considerably by a term proportional to τ^2. This broadening vanishes for identical charge to mass ratios of colliding particles. At the same time the momentum of the distribution function becomes retarded by the electric field. This effect is sometimes called the intracollisional field effect.

IV. FIELD EFFECTS ON CURRENT

We are now interested in corrections to the particle flux, and therefore obtain from Eq. (48) the balance equation for the momentum analog to Eq. (3),

$$\frac{\partial}{\partial t}\langle \mathbf{k}_a\rangle - n_a e_a \mathbf{E} = \sum_b \langle \mathbf{k}_a I_B^{ab}\rangle. \tag{51}$$

Here we search for the relaxation field (5) which will be represented as a renormalization of the external field **E** similar to the Debye-Onsager relaxation field in the theory of electrolyte transport [14–16]. This effect is a result of the deformation of the two-particle correlation function by an applied electric field.

To proceed we assume some important restrictions on the distribution functions. First, we assume a nondegenerate situation, such that Pauli blocking effects can be neglected. Second, to calculate the current for a quasistationary plasma we choose Maxwellian distributions analogous to Eq. (10),

$$f_i(p) = \frac{n_i}{2s_i+1}\lambda_i^3 \exp\left\{-\frac{p^2}{2m_iT_i}\right\}, \tag{52}$$

with the thermal wavelength $\lambda_i^2 = 2\pi\hbar^2/(m_iT_i)$, spin s_i, and partial temperature T_i for species i, which can be quite different, e.g., in a two-component system.

A. Statically screened result

Before we present the result for the dynamically screened approximation we want to give the static result. The momentum conservation in Eq. (48) can be carried out and we get for the relaxation field the result

$$n_a e_a \frac{\delta E}{E}\mathbf{E} = \sum_b \frac{2s_as_b}{\hbar^2(2\pi\hbar)^9}\int d\mathbf{k}d\mathbf{q}d\mathbf{Q}\; f_b(\mathbf{Q})f_a(\mathbf{k}) \times V^2(q)\mathbf{q}\int_0^\infty d\tau \cos\left[\left(-\frac{q^2}{2\mu\hbar}-\frac{\mathbf{k}\cdot\mathbf{q}}{m_a\hbar}+\frac{\mathbf{q}\cdot\mathbf{Q}}{m_b\hbar}\right)\tau + \frac{\mathbf{E}\cdot\mathbf{q}}{2\hbar}\left(\frac{e_b}{m_b}-\frac{e_a}{m_a}\right)\tau^2\right] \tag{53}$$

with the reduced mass $\mu^{-1} = 1/m_a + 1/m_b$. The angular integrations can be carried out trivially and we get

$$n_a e_a \frac{\delta E}{E}\mathbf{E} = \frac{\mathbf{E}}{E}\sum_b I_1,$$

$$I_1 = \frac{1}{\hbar^{11}4\pi^6}\int dq q^3 V^2(q)\int_0^\infty d\tau \,\mathrm{js}\left(\frac{Eq}{2\hbar}\left[\frac{e_b}{m_b}-\frac{e_a}{m_a}\right]\tau^2\right) \times \sin\left(\frac{q^2\tau}{2\mu\hbar}\right) I_2[a]I_2[b] \tag{54}$$

with $\mathrm{js}(x) = (x\cos x - \sin x)/x^2$. The two integrals over the distribution functions I_2 can be performed with the result

$$I_2[a] = \frac{\hbar m_a(2s_a+1)}{q\tau}\int_0^\infty dk k\, f_a(k)\sin\left(\frac{kq\tau}{m_a\hbar}\right) = 2\hbar^3 n_a \pi^2 e^{-q^2\tau^2T_a/2\hbar^2m_a} \tag{55}$$

and correspondingly $I_2[b]$. We now introduce the new variables

$$q = 2y\sqrt{\mu T_{ab}},$$
$$t = \frac{2T_{ab}\tau}{\hbar},$$
$$T_{ab} = \frac{1}{2}\left(\frac{m_b}{m_a+m_b}T_a + \frac{m_a}{m_a+m_b}T_b\right), \tag{56}$$
$$e = \frac{\hbar\sqrt{\mu}E}{4T_{ab}^{3/2}}\left[\frac{e_b}{m_b}-\frac{e_a}{m_a}\right],$$

and obtain

$$I_1 = \frac{8n_an_b\mu^2T_{ab}}{\pi^2\hbar^4}\int_0^\infty dy y^3\; V^2(2y\sqrt{\mu T_{ab}}) \times \int_0^\infty dt\,\mathrm{js}(yt^2e)\sin(y^2t)e^{-y^2t^2}. \tag{57}$$

Using the screened Debye potential (49) we finally obtain

$$I_1 = \frac{8n_an_be_a^2e_b^2}{T_{ab}}I_3,$$
$$I_3 = \int_0^\infty dz \frac{z^3}{(z^2+1)^2}\int_0^\infty dl\,\mathrm{js}(xzl^2)\frac{\sin(z^2l\zeta)}{\zeta}e^{-z^2l^2}. \tag{58}$$

Here we used $y = z\zeta$ and $l = t\zeta$ with the quantum parameter

$$\zeta^2=\frac{\hbar^2\kappa^2}{4\mu T_{ab}} \tag{59}$$

and the classical field parameter

$$x=\frac{e}{\zeta}=\frac{E}{2T_{ab}\kappa}\left(\frac{m_a}{m_a+m_b}e_b-\frac{m_b}{m_a+m_b}e_a\right). \tag{60}$$

With this form (58) we have given an extremely useful representation because the field effects, contained in x, are separated from the quantum effects, which are contained in ζ. The integral in Eq. (58) can be performed analytically in the classical limit $\zeta\to 0$. For the more general quantum case with arbitrary ζ the linear and cubic field effects can be given analytically and are discussed in [17]. We will not discuss them here.

Performing the classical limit $\zeta\to 0$ one obtains from Eq. (58) that [18,17]

$$\begin{aligned} I_{3c}&=-\frac{\pi x}{24}F(|x|),\\ F(x)&=-\frac{3}{x^2}\left[3-x+\frac{1}{1+x}-\frac{4}{x}\ln(1+x)\right]. \end{aligned} \tag{61}$$

Introducing the classical result Eq. (61) into Eq. (58) we find from Eqs. (54) and (51) the following relaxation field:

$$\frac{\partial}{\partial t}\langle\mathbf{k}_a\rangle-n_a e_a\mathbf{E}\left(1+\frac{\delta E_a}{E}\right)=n_a e_a\mathbf{J}\,R(E) \tag{62}$$

with

$$\frac{\delta E_a}{E}=-\frac{e_a\pi}{6\kappa}\sum_b\frac{4n_b e_b^2}{\mu_{ab}}\frac{e_b/m_b-e_a/m_a}{(T_b/m_b+T_a/m_a)^2}F(|x|) \tag{63}$$

and x from Eq. (60). We see that for a plasma consisting of particles with equal charge to mass ratios, no relaxation field appears. The link to the known Debye-Onsager relaxation effect can be found if we assume that we have a plasma consisting of electrons ($m_e, e_e=e$) and ions with charge $e_i=eZ$ and temperatures $T_e=T_i=T$. Then Eq. (63) reduces to

$$\begin{aligned} \frac{\delta E_a}{E}&=-\frac{\kappa e_a^2}{6T}\frac{Z[1+(m_e/m_i)Z]}{(1+Z)(1+m_e/m_i)}F\left(\frac{eE}{T\kappa}\frac{Z[1+(m_e/m_i)Z]}{1+m_e/m_i}\right)\\ &=-\frac{e^2\kappa_e}{6T}\begin{cases}\frac{1}{2}+o(E)\\ \frac{3\kappa T}{2eE}+o(1/E)^2\end{cases}\quad\text{for } Z=1. \end{aligned} \tag{64}$$

This formula together with the general form (63) is the main result of this section. It gives the classical relaxation effect for the statically screened approximation up to any field strength and represents a result beyond linear response. We see that in the case of singly charged heavy ions the Debye result (1) is underestimated by a factor of 2.

B. Dynamically screened result

The calculation of the current with the collision integral for dynamically screened potentials (46) can be performed analytically as well. For the quasistationary condition we can calculate the frequency integral in Eq. (46) analytically using the identity [44] for the classical limit $o(\hbar)$,

$$\int\frac{d\omega}{2\pi}\frac{H(\omega)}{\omega}\mathrm{Im}\,\mathcal{E}^{-1}(q,\omega)=\frac{H(0)}{2}\mathrm{Re}\left(1-\frac{1}{\mathcal{E}(q,0)}\right) \tag{65}$$

where we set $H(\omega)=\omega/\mathrm{Im}\,\mathcal{E}$. We will employ only classical screening. The quantum result for screening is more involved and not yet analytically integrable.

Observing that for the dielectric function Eq. (44) together with Eq. (52) the following holds:

$$\lim_{\omega\to 0}\frac{\omega}{\mathrm{Im}\,\mathcal{E}(q,\omega)}=\frac{q^3}{\sqrt{\pi}\hbar^3}\left(\sum_b\frac{\kappa_b^2}{v_b}\right)^{-1} \tag{66}$$

with the partial screening length $\kappa_b^2=4\pi e_b^2 n_b/T_b$ and the partial thermal velocity $v_b^2=2T_b/m_b$, we obtain for the current (54) after similar integrations as in Sec. IV A, instead of Eq. (58),

$$\begin{aligned} I_1^{\mathrm{dyn}}&=\frac{8\kappa^2 e_a^2 e_b^2 n_a n_b\sqrt{m_a m_b}}{\sqrt{\pi\mu_{ab}TT_aT_b}\sum_c\kappa_c^2/v_c}I_3^{\mathrm{dyn}},\\ I_3^{\mathrm{dyn}}&=\int_0^\infty dz\frac{z^2}{1+z^2}\int_{-1}^1 dx x\int_0^\infty dl dl_1 e^{-z^2(l^2+l_1^2)}\\ &\quad\times\frac{1}{\zeta}\cos[M_b\zeta l z^2+Bzl^2x]\cos[M_a\zeta l_1 z^2-Azl_1^2x]. \end{aligned} \tag{67}$$

Here we used the same dimensionless variables as in Sec. IV A and the quantum parameter (59). Further, we abbreviated $A=e_aE/\kappa T_a$, $B=e_bE/\kappa T_b$, $M_a=\sqrt{2\mu T/m_aT_a}$, $M_b=\sqrt{2\mu T/m_bT_b}$.

We wish to remark that we neglect any field dependence of the screening $\mathcal{E}$ itself here. As presented in [39] a field-dependent screening function can be derived. However, this field dependence starts quadratically and will not be considered in this work.

The classical limit of Eq. (67) can be performed again by letting $\zeta\to 0$. We obtain

$$I_3^{\mathrm{dyn}}=\frac{1}{2}AM_aI[|A|,|B|]-(a\leftrightarrow b) \tag{68}$$

with the remaining three-dimensional integral

$$\begin{aligned} I[A,B]&=\int_0^\infty dz\frac{z^3}{z^2+1}\int_{-1}^1 dx\frac{x^2}{A^2x^2+z^2}\\ &\quad\times\int_0^\infty dl e^{-z^2l^2}\cos(Bzl^2x). \end{aligned} \tag{69}$$

1. Linear response

The linear response can be read off directly from Eq. (68) and is given by $I[0,0]$ of Eq. (69). We obtain

$$I_3^{\rm dyn}=\frac{\pi^{3/2}}{12}(M_aA-M_bB) \tag{70}$$

and the linear relaxation field (63) takes the form

$$\frac{\delta E^{\rm dyn}}{E}=\frac{4e\pi\kappa}{3\sum_c \kappa_c^2\sqrt{m_c/T_c}}\sum_b n_b e_b^2\sqrt{\frac{m_am_b}{T_aT_b}} \times\left(\frac{e_a}{T_a^{3/2}\sqrt{m_a}}-\frac{e_b}{T_b^{3/2}\sqrt{m_b}}\right)+o(E). \tag{71}$$

The difference from Eq. (63) becomes more evident if we consider again only electrons and ions with equal temperature,

$$\frac{\delta E^{\rm dyn}}{E}=-\frac{\kappa e^2}{6T}\frac{2Z(1+\sqrt{m_e/m_i}Z)}{(Z+\sqrt{m_e/m_i})}+o(E). \tag{72}$$

The differences from Eq. (64) are obvious in the different mass dependence. This result overestimates the Debye result by a factor of 2.

2. Complete classical result

Now we are able to present a complete field dependence beyond linear response. The integral (69) can be done analytically, as sketched in the Appendix. The result reads

$$I[A,B]=\frac{\pi^{3/2}}{6}\mathcal{I}[A,B],$$

$$\mathcal{I}[A,B]=\frac{3}{2A^3}\left[\frac{4A(1-\sqrt{1+B})}{B}+\frac{A^2+\ln(1-A^2)}{\sqrt{1+B/A}}+2\left(\frac{\operatorname{arctanh}(1/\sqrt{1-B/A})-\operatorname{arctanh}(\sqrt{1+B}/\sqrt{1-B/A})}{\sqrt{1-B/A}}\right.\right.$$
$$\left.\left.+\frac{-\operatorname{arctanh}(1/\sqrt{1+B/A})+\operatorname{arctanh}(\sqrt{1+B}/\sqrt{1+B/A})}{\sqrt{1+B/A}}\right)\right]. \tag{73}$$

We obtain for Eq. (71)

$$\frac{\delta E_a^{\rm dyn}}{E}=\frac{4e_a\pi\kappa}{3\sum_c \kappa_c^2\sqrt{m_c/T_c}}\sum_b n_b e_b^2\sqrt{\frac{m_am_b}{T_aT_b}} \times\left(\frac{e_a}{T_a^{3/2}\sqrt{m_a}}\mathcal{I}[A,B]-\frac{e_b}{T_b^{3/2}\sqrt{m_b}}\mathcal{I}[B,A]\right). \tag{74}$$

Expanding (73) in powers of E we recover (71). Once more we choose the case of electrons and ions with equal temperature and obtain

$$\frac{\delta E^{\rm dyn}}{E}=-\frac{\kappa e^2}{6\epsilon_0 T}\frac{2Z(\mathcal{I}[A,B]+\sqrt{m_e/m_i}Z\mathcal{I}[B,A])}{(Z+\sqrt{m_e/m_i})}. \tag{75}$$

For singly charged ions and big mass differences we can further simplify to

$$\frac{\delta E^{\rm dyn}}{E}=-\frac{\kappa e^2}{6T}\mathcal{F}\left[\frac{eE}{\kappa T}\right]=-\frac{e^2\kappa_e}{6T}\times\begin{cases}2+o(E)\\ \dfrac{3\kappa T}{\sqrt{2}eE}+o(1/E)^2,\end{cases} \tag{76}$$

$$\mathcal{F}[x]=\frac{3}{x^3}\left\{\frac{2[-2x+3(-1+\sqrt{1+x})]}{\sqrt{1+x}}+\sqrt{2}\left[-\operatorname{arctanh}\left(\frac{1}{\sqrt{2}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)\right]+\frac{x^2+\ln(1-x^2)}{\sqrt{2}}\right\}$$
$$=2+o(E).$$

This result will be compared with the statically screened result (64) and the hydrodynamical result (24) in Sec. IV C. Here we remark that the Debye result is twice overestimated here.

C. Thermally averaged dynamically screened result

We will now give an approximate treatment of the dynamical screening used in [8]. This approximation consists of the replacement of the dynamical screening in the collision integral (46), which is $\overline{\mathcal{E}(\omega,q)^{-2}}$, by $[1+\kappa^2V_{aa}(q)/4\pi]^{-1}$. This represents a thermal averaging [44] of $\mathcal{E}^{-2}$, which can be proved easily with the help of Eq. (65). We obtain the relaxation effect of Eqs. (63) and (64) but with a different field function F,

$$F^{\rm dyn}(x)=-\frac{3}{x^2}\left[2-x-\frac{2}{x}\ln(1+x)\right]=\begin{cases}2+o(x)\\ \dfrac{3}{x}+o(1/x)^2.\end{cases} \tag{77}$$

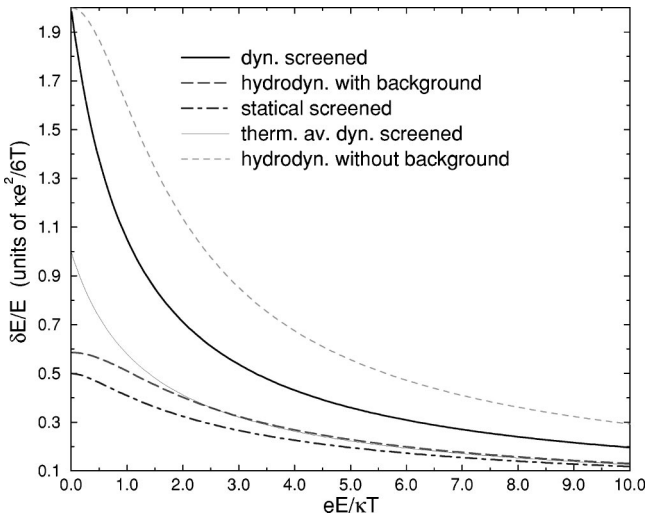

FIG. 1. The nonlinear Debye-Onsager relaxation effect vs scaled electric field for an electron and singly charged ion system. The hydrodynamical approximation (24) leads to the Onsager result (2) for small field strength $2-\sqrt{2}$. The statically screened result (63) or (64) leads to half the Debye result (1). The thermally averaged approximation of the dynamical screening (77) leads to the Debye result while the full dynamically screened approximation (76) leads to twice the Debye result. Also, the hydrodynamical result (28) without background leads to twice the Debye result.

Therefore the relaxation effect (64) in linear response for singly charged ions takes the form of Eq. (1) and is twice the static screened result (64) and half of the dynamical screened result (76).

As we see from Fig. 1 the different approximations lead to very different results. The statically screened result (64) underestimates the Debye result by a factor 2, which is corrected by the thermally averaged treatment of screening. If we calculate instead the complete dynamical screened result (72) or (75), we obtain twice the Debye result (1) and the thermally averaged screened result. However, there is a completely different charge dependence. We have to observe that the perfectly symmetric treatment of screening does not reproduce the hydrodynamical result, which is the Onsager result (2) for linear response.

D. Asymmetric dynamical screened result

We want now to proceed and ask under what assumptions the Onsager result (2) might be reproduced. Following the results we saw from the hierarchy we consequently have to treat the electrons (species a) and ions (all other species) asymmetrically. This we will perform in the same spirit as Onsager in that the ions have to be treated dynamically (as before) but the electrons are screened statically.

This means we consider not the bare Coulomb potential but a statically screened Debye potential for species a. The ions (all other species) will then form the dynamical screening. In comparison with the preceding section we can perform all steps analogously except for two modifications, Eqs. (80) and (83). First we observe that instead of Eq. (66) we have now

$$\lim_{\omega\to 0}\frac{\omega}{\mathrm{Im}\,\mathcal{E}(q,\omega)}=\frac{qv_b(q^2+\hbar^2\kappa_e^2)}{\sqrt{\pi}\hbar^3\kappa_b^2}, \tag{78}$$

which leads to a replacement of the sum

$$\sum_c \frac{\kappa_c^2}{v_c}\to\frac{\kappa^2}{v_i} \tag{79}$$

in the factor of Eqs. (67) and (74). This leads in the limit of big mass differences to a factor in Eqs. (72) and (75), respectively, of

$$\frac{Z}{1+Z}\quad\text{(modification I)}. \tag{80}$$

The second modification is that in Eq. (67) one has to replace

$$\frac{z^2}{1+z^2}\to\frac{z^2}{1+z^2}\frac{z^2}{q+z^2}=\frac{q}{q-1}\frac{z^2}{q+z^2}-\frac{1}{q-1}\frac{z^2}{1+z^2} \tag{81}$$

with

$$q=\frac{\kappa_a^2}{\kappa^2}. \tag{82}$$

This shows that in the end results (74) and (75) have to be changed to

$$\mathcal{I}[A,B]\to\sqrt{\frac{q}{q-1}}\mathcal{I}\left[\frac{A}{\sqrt{q}},\frac{B}{\sqrt{q}}\right]-\frac{1}{q-1}\mathcal{I}[A,B]\quad\text{(modification II)}. \tag{83}$$

In particular, we obtain for the linear response result (72), where for an electron-ion plasma $q=1/(Z+1)$,

$$\frac{\delta E^{\mathrm{asy}}}{E}=\frac{\delta E^{\mathrm{dyn}}}{E}\frac{Zq}{\sqrt{q}+1}=-\frac{\kappa e^2}{3T}\frac{Zq}{\sqrt{q}+1}+o(E), \tag{84}$$

which agrees with Eq. (21) if we consider that the mobilities are very different, $b_i/b_e\to 0$ in Eq. (22).

We obtain the same result from the thermally averaged result (61), since there appears no such function as Eq. (78) and therefore the modification I of Eq. (80) does not apply, but solely the modification II of Eq. (83). We therefore obtain Eq. (64) but

$$F^{\mathrm{dyn}}_{\mathrm{asy}}(x)=2F^{\mathrm{dyn}}(x)-\sqrt{2}F^{\mathrm{dyn}}(\sqrt{2}x)=\begin{cases}2-\sqrt{2}+o(x)\\ \dfrac{3}{2x}+o(1/x)^2\end{cases} \tag{85}$$

with F^{dyn} of Eq. (77). The linear response then leads exactly to the same result as from the dynamical screening (84), i.e., the Onsager result with the same charge dependence.

The fact that we reproduce the classical Onsager result with the same charge dependence can be considered as very satisfactory, the more so since we have seen how many different considerations are possible. Please note that the special case $Z=1$ could lead occasionally to a seeming agreement between different treatments. We think that the charge dependence discriminates among different treatments. In Fig. 2 we see that the asymmetrical screened result (75) with (83) approaches the hydrodynamical or Onsager result (2) rather

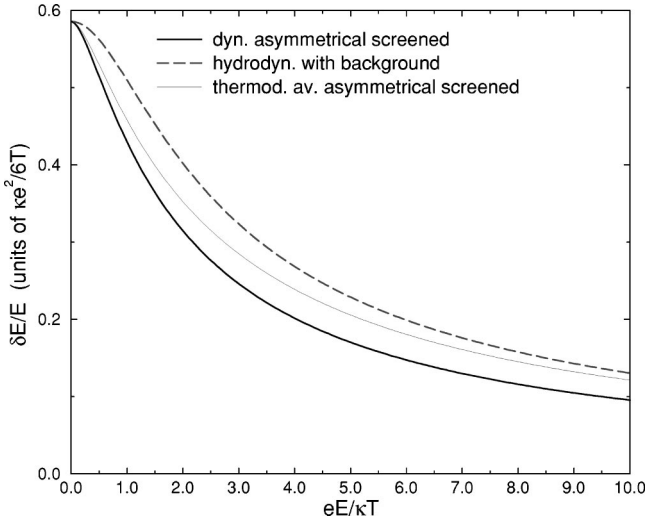

FIG. 2. The different asymmetric screened approximations for the relaxation effect versus field strength. The hydrodynamical result (24) is compared with the thermally averaged asymmetric screened result of (85) as well as the asymmetric screened one of (83).

well for small fields while it is too low at high fields. On the other hand, the thermally averaged symmetrical screened result (85) agrees with the hydrodynamical approximation (24) in the low and high field limits. Why the hydrodynamical result cannot be reproduced completely within the kinetic theory still remains a puzzle. Probably the remaining difference is due to the neglect of the field effect on the screening itself [39].

V. RANGE OF APPLICABILITY

During the derivation of the quantum kinetic equations gradient approximations have been assumed that restricted the spatial gradients of the system. Here we want to discuss up to what field strength this assumption is justified.

The electric field is limited to values $x \ll 1$ for x from Eq. (60). This can be deduced from the expression for the dynamical screened result (76). The expression has a removable singularity at $x=1$. Therefore we see a smooth curve. Nevertheless, this is the field strength where something is happening. For equal masses and temperatures of plasma components this condition translates into

$$E < \frac{\kappa T}{e}. \tag{86}$$

We interpret the occurrence of such a singular point as meaning that no thermal distributions pertain to the system. Thus we have to take into account nonthermal field-dependent distributions, which have been employed to study nonlinear conductivity [45–47].

The condition (86) allows for different physical interpretations. Within the picture of the screening cloud we can rewrite Eq. (86) as

$$eE < m \frac{v_{\rm th}^2}{r_D}. \tag{87}$$

This means that a particle moving on the radius of the screening cloud $r_D = 1/\kappa$ with thermal velocity $v_{\rm th}^2 = T/m$ should not be pulled away by the acting field force. We can discuss this limit also via the energy density, which can be reached in a system by the applied field. We can reformulate the condition (86) once more to find, equivalently,

$$\frac{E^2}{4\pi} < nT. \tag{88}$$

This means that we have essentially nonthermal effects, to be expected if the energy density of the field becomes comparable with the thermal energy density.

The validity criterion (86) can now be used to check the weak space inhomogeneity that has been assumed during our calculation. Quasiequilibrium in charged systems with external fields can be assumed only if the field current is accompanied by an equivalent diffusion current,

$$j_{\rm field} = e\mu E n = -j_{\rm diff} = eD\frac{dn}{dx}. \tag{89}$$

Using the Einstein condition $\mu = eD/T$ one gets

$$eE = T\frac{1}{n}\frac{dn}{dx}. \tag{90}$$

Combining this elementary consideration with our condition (86) we obtain a limitation for space gradients

$$\frac{dn}{d(\kappa x)} < n, \tag{91}$$

where our treatment of field effects and local equilibrium is applicable.

VI. SUMMARY

The nonlinear relaxation field of a charged system under the influence of high electric fields is investigated. The local equilibrium or hydrodynamical approach starting from the classical BBGKY hierarchy is compared with the results from the quantum kinetic equations beyond linear response.

We come to the same conclusion considering the hydrodynamical approximation or the kinetic theory that a perfectly symmetric two-component plasma will lead to a different relaxation effect from the case where we consider the moving charge asymmetrically from the screening surroundings. In the hydrodynamic approach this was achieved by friction with a background; in the kinetic approach we realized it due to asymmetric screening. Within this asymmetric treatment the limit of a one-component plasma, which would be to set the ion charge to $Z=-1$, leads to a nonvanishing finite quantity. In contrast, in the perfectly symmetrical treatment this limit vanishes in that the relaxation field vanishes, as it should. The perfectly symmetrical treatment of species in the system leads to twice the Debye result, different from the Onsager result in linear response.

Since the situation of small friction with a background or disturbance is certainly more likely to be found in nature than the case of a mathematically pure two-component plasma, we consider the hydrodynamical approximation

leading to the Onsager result as more realistic. However, one should keep in mind that this Onsager result distinguishes and treats the two components differently, which results in the fact that we cannot achieve from the Onsager result the limit of a one-component plasma that it is possible to obtain from the alternatively presented pure two-component plasma. The difference between these two results, the pure two-component one and that including friction, illustrates a symmetry breaking since there is no continuous transition between the results if the disturbance or friction is made infinitesimally small. This underlines the subtlety of handling two-particle correlations.

Different approximations of the kinetic approaches, on the other hand, are compared and discussed as well. The symmetrical and asymmetrical treatment of species leads to results which cannot be transferred into each other even for infinitesimal small symmetry breaking analogously to the hydrodynamical approach. We found agreement for linear response with the hydrodynamical approach even for arbitrary charge dependences. This suggests possible occasional agreement of former treatments that used singly charged ions. For higher field strengths beyond linear response, there appear minor differences, which are probably due to the neglect of the field-dependent screening itself. The thermally averaged approximation of screening has the advantage of agreeing for low and high fields with the hydrodynamical local equilibrium approach.

ACKNOWLEDGMENTS

The author wants to warmly thank Jens Ortner who contributed the hydrodynamical approach with background. Also, many comments regarding the manuscript are gratefully acknowledged. Last but not least, Gerd Röpke is thanked for stimulating intellectual disagreements that provoked this work.

APPENDIX: AN INTEGRAL

Here we calculate the integral (69)

$$I[a,b]=\int_0^\infty dz\,\frac{z^3}{z^2+1}\int_{-1}^{1}dx\,\frac{x^2}{a^2x^2+z^2}\times\int_0^\infty dl\,e^{-z^2l^2}\cos(bzl^2x). \tag{A1}$$

The variable substitutions $l\to p$ by $p=\sqrt{z}l$, $z\to y$ by $z=yx$, and $p\to e$ by $p\sqrt{x}=e$ leads to

$$I[a,b]=2\int_0^1 dx\int_0^\infty dy\,\frac{y^{5/2}}{y^2x^2+1}\,\frac{x^3}{a^2+y^2}\int_0^\infty de\,e^{-ye^2}\cos(be^2)$$
$$=\int_0^\infty dy\,\frac{y^{1/2}}{a^2+y^2}\left(1-\frac{\ln(1+y^2)}{y^2}\right)\times\int_0^\infty de\,e^{-ye^2}\cos(be^2), \tag{A2}$$

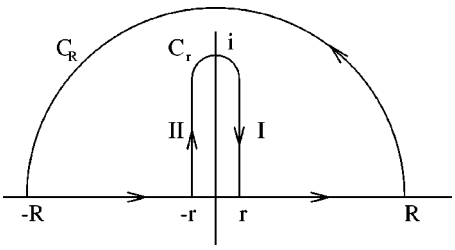

FIG. 3. The complex integration path for the integral (A6).

where the trivial x integration has been carried out. The variable substitution $e\to q$ by $\sqrt{y}e=q$ and $y\to z$ by $y=1/z$ leads to

$$I[a,b]=\frac{1}{a^2}\int_0^\infty dq\,e^{-q^2}\int_0^\infty dy\,\cos(bq^2z)\,\frac{1-z^2\ln(1+1/z^2)}{z^2+1/a^2}. \tag{A3}$$

Now we proceed and use an integral calculated in the next subsection:

$$\int_{-\infty}^{\infty}dy\,e^{icy}\,\frac{1-y^2\ln(1+1/y^2)}{y^2+1/a^2}=2\pi\int_0^1 dx\,x^2\,\frac{e^{-cx}}{1/a^2-x^2}+a\pi e^{-c/a}\left(1+\frac{\ln(1-a^2)}{a^2}\right) \tag{A4}$$

to obtain for Eq. (A3)

$$I[a,b]=\frac{\pi^{3/2}}{4a}\,\frac{1+[\ln(1-a^2)]/a^2}{\sqrt{b/a}}+\frac{\pi^{3/2}}{2a^2}\int_0^1 dx\,\frac{x^2}{(1/a^2-x^2)\sqrt{1+bx}}. \tag{A5}$$

The last integral is trivial and we end up with Eq. (73).

1. Another integral

Our task now remains to solve the integral

$$I=\int_{-\infty}^{\infty}dy\,e^{icy}\,\frac{1-y^2\ln(1+1/y^2)}{y^2+1/a^2}. \tag{A6}$$

Because the complex function $\ln(1+1/y^2)$ has a cut from $(0,i)$ we perform the integration along the path as depicted in Fig. 3 and write

$$\int_R^{-r}+\int_r^R+C_R+C_r+\int_I+\int_{II}=2\pi i\,\mathrm{Res}\left[\frac{1-y^2\ln(1+1/y^2)}{y^2+1/a^2},i/a\right]=\pi a e^{-c/a}\left(1+\frac{\ln(1-a^2)}{a^2}\right). \tag{A7}$$

It is now easy to prove that in the limit $r\to 0$ and $R\to\infty$ the integration parts C vanish. Since the first two parts of Eq. (A7) represent just the desired integral I we have to calculate

$$\int_I + \int_{II} = \int_{i+r}^{r} dy e^{icy} \frac{1-y^2\ln(1+1/y^2)}{y^2+1/a^2} + \int_{-r}^{i-r} dy e^{icy} \frac{1-y^2\ln(1+1/y^2)+2\pi i}{y^2+1/a^2} = -2\pi \int_0^1 dx \frac{x^2 e^{-cx}}{1/a^2-x^2}. \tag{A8}$$

Using Eqs. (A8) and (A7) we obtain just Eq. (A4).

[1] H. Haug and A.P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors* (Springer, Berlin, 1996).
[2] W. Theobald, R. Häßner, C. Wülker, and R. Sauerbrey, Phys. Rev. Lett. **77**, 298 (1996).
[3] W.D. Kraeft, D. Kremp, W. Ebeling, and G. Röpke, *Quantum Statistics of Charged Particle Systems* (Akademie Verlag, Berlin, 1986).
[4] Y. Kluger, J.M. Eisenberg, and B. Svetitsky, Phys. Rev. D **45**, 4659 (1992).
[5] B.B. Kadomtsev, Zh. Éksp. Teor. Fiz. **33**, 151 (1958) [Sov. Phys. JETP **33**, 117 (1958)].
[6] Y.L. Klimontovich and W. Ebeling, Zh. Éksp. Teor. Fiz. **63**, 905 (1972) [Sov. Phys. JETP **36**, 476 (1973)].
[7] W. Ebeling, Ann. Phys. (Leipzig) **33**, 5 (1976).
[8] W. Ebeling and G. Röpke, Ann. Phys. (Leipzig) **36**, 429 (1979).
[9] G. Röpke, Phys. Rev. A **38**, 3001 (1988).
[10] K. Morawetz and D. Kremp, Phys. Lett. A **173**, 317 (1993).
[11] A. Esser and G. Röpke, Phys. Rev. E **58**, 2446 (1998).
[12] P. Debye and E. Hückel, Phys. Z. **15**, 305 (1923).
[13] L. Onsager, Phys. Z. **8**, 277 (1927).
[14] H. Falkenhagen, *Elektrolyte* (S. Hirzel Verlag, Leipzig, 1953).
[15] H. Falkenhagen, W. Ebeling, and W.D. Kraeft, in *Ionic Interaction*, edited by S. Petrucci (Academic Press, New York, 1971), Chap. 1, p. 1.
[16] D. Kremp, D. Kraeft, and W. Ebeling, Ann. Phys. (Leipzig) **18**, 246 (1966).
[17] K. Morawetz, Contrib. Plasma Phys. **37**, 195 (1997); **37**, 4(E) (1997).
[18] J. Ortner, Phys. Rev. E **56**, 6193 (1997).
[19] N.N. Bogoliubov, J. Phys. (Moscow) **10**, 256 (1946); Translated in *Studies in Statistical Mechanics*, Vol. 1, edited by D. de Boer and G.E. Uhlenbeck (North-Holland, Amsterdam, 1962).
[20] Y.L. Klimontovich and W. Ebeling, Zh. Éksp. Teor. Fiz. **43**, 146 (1962) [Sov. Phys. JETP **16**, 104 (1963)].
[21] P. Danielewicz, Ann. Phys. (N.Y.) **152**, 239 (1984).
[22] V. Špička and P. Lipavský, Phys. Rev. Lett. **73**, 3439 (1994).
[23] V. Špička and P. Lipavský, Phys. Rev. B **52**, 14 615 (1995).
[24] K. Morawetz, Phys. Lett. A **199**, 241 (1995).
[25] P. Lipavský, F.S. Khan, F. Abdolsalami, and J.W. Wilkins, Phys. Rev. B **43**, 4885 (1991).
[26] L.P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).
[27] L.V. Keldysh, Zh. Éksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20** , 1018 (1965)].
[28] C. Itzykson and J.B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1990).
[29] R. Bertoncini and A.P. Jauho, Phys. Rev. B **44**, 3655 (1991).
[30] K. Morawetz and G. Röpke, Z. Phys. A **355**, 287 (1996).
[31] A.P. Jauho and J.W. Wilkins, Phys. Rev. B **29**, 1919 (1984).
[32] G.D. Mahan, Phys. Rep. **145**, 251 (1987).
[33] P. Lipavský, V. Špička, and B. Velický, Phys. Rev. B **34**, 6933 (1986).
[34] A.P. Jauho, in *Quantum Transport in Semiconductors*, edited by D. Ferry and C. Jacoboni (Plenum Press, New York, 1991), Chap. 7.
[35] F.S. Khan, J.H. Davies, and J.W. Wilkins, Phys. Rev. B **36**, 2578 (1987).
[36] M. Abramowitz and I.A. Stegun, *Pocketbook of Mathematical Functions* (Verlag Harri Deutsch, Frankfurt/Main, 1984).
[37] K. Morawetz, Phys. Rev. E **50**, 4625 (1994).
[38] R. Bertoncini, A.M. Kriman, and D.K. Ferry, Phys. Rev. B **40**, 3371 (1989).
[39] K. Morawetz and A.P. Jauho, Phys. Rev. E **50**, 474 (1994).
[40] I.B. Levinson, Fiz. Tverd. Tela (Leningrad) **6**, 2113 (1965) [Sov. Phys. Solid State **6**, 1665 (1965)].
[41] I.B. Levinson, Zh. Éksp. Teor. Fiz. **57**, 660 (1969) [Sov. Phys. JETP **30**, 362 (1970)].
[42] K. Morawetz, P. Lipavský, and V. Špička, Phys. Rev. B (to be published) (e-print cond-mat/0005287).
[43] S.K. Sarker, J.H. Davies, F.S. Khan, and J.W. Wilkins, Phys. Rev. B **33**, 7263 (1986).
[44] Y.L. Klimontovich, *Kinetic Theory of Nonideal Gases and Nonideal Plasmas* (Academic Press, New York, 1975).
[45] K. Morawetz, M. Schlanges, and D. Kremp, Phys. Rev. E **48**, 2980 (1993).
[46] D. Kremp, K. Morawetz, M. Schlanges, and V. Rietz, Phys. Rev. E **47**, 635 (1993).
[47] K. Morawetz and D. Kremp, Phys. Plasmas **1**, 225 (1994).